NONSTEADY RADIAL EXPANSION OF A GAS INTO A FLOODED SPACE FROM A SUDDENLY TURNED ON STEADY SOURCE

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The problem of the radial expansion of a gas into a medium of low pressure from a suddenly turned on steady source with $M \geq 1$ is examined in the report. The examination concerns mainly the unsteady flow of gas in the initial stage of expansion. The laws of motion of the surface of a powerful explosion, which determine the structure of the region of flow, are established for the initial stage of expansion with the help of a simple approximate solution of the problem. A method is proposed and a numerical solution of the problem is obtained for a viscous thermally conducting gas in a general formulation.

The steady supersonic flows of a heated gas in nozzles and in free jets beyond underexpanded nozzles are widely used in laboratory practice for performing physical and aerodynamic studies. In this case the heating of the gas in the forechamber of the gasdynamic source is often accomplished with a pulsed electric discharge or shock compression of the gas [1, 2]. In these cases the discharge of the gas from the forechamber into the expanding part of the nozzle or into free space begins as a result of the sharp increase in pressure in the forechamber. A steady mode of flow is rapidly established in the critical cross section of the nozzle and the further development of the gas flow in the expanding part of the nozzle or in free space now takes place in the presence of a steady source of gas in the critical cross section of the nozzle with the parameters p_*, T_*, and u_* corresponding to the parameters p_0 and T_0 of the stagnant gas in the forechamber. If p_0 and T_0 remain constant for a long enough time then the corresponding steady flow is established in the nozzle or in the jet. In an actual case [1, 2] the time within which p_0 and T_0 can be taken as constant is small. Therefore it is important to know how the steady flow is formed, especially in the central region of the stream usually used for performing studies.

The flow of gas in the central region of jets escaping from an axially symmetric or flat nozzle with a strongly underexpanded stream, as well as the flow in axially symmetric or flat nozzles with a rectangular contour (with the condition of neglecting friction at the nozzle walls), is close to radial flow (with spherical or cylindrical symmetry, respectively) [1-3]. The idealization of the original problem leads to the problem of the radial (nonsteady) expansion of a gas from a suddenly turned on steady source.

The problem was solved in [1] for the case of expansion of a gas into a vacuum (in application to flow in nozzles). The asymptotic form of flow as $t \to \infty$ for the expansion of a gas into a flooded space was studied in [3] (in application to jet flows). In [1, 3] the analysis was conducted in the framework of the theory of an ideal fluid.

1. Statement of the Problem

Let a spherical or cylindrical surface of radius $r_1 \neq 0$ be defined in an infinite volume of a quiescent gas with known parameters of state (p_∞, T_∞). It is required to determine the development of the flow of the gas in the region (r_1, ∞) with the course of time if at some moment $t = t_1$ the parameters of the gas at the surface $r = r_1$ suddenly acquire the fixed values $p_1 > p_\infty$, T_1, $u_1 \geq 0$ ($M_1 \geq 1$) which do not vary with time.

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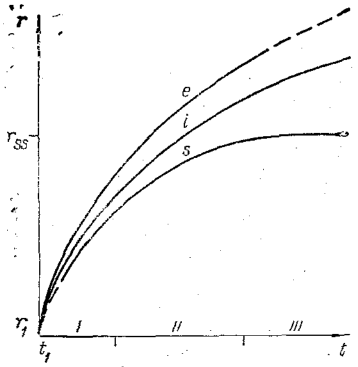

Fig. 1

2. General Pattern of Flow

Let us follow the process of development of the flow during the radial expansion of a gas (ideal fluid) into a flooded space from a suddenly turned on steady source with $M_1 \geq 1$. In this case we will partly use the results of [1, 3]. The qualitative pattern of flow is analogous for $\nu = 1$ and $\nu = 2$.

The paths of motion of the surfaces of a powerful explosion which determine the structure of the region of flow are shown in Fig. 1. The contact surface separating the gas of the source and the surrounding gas, the shock wave in the surrounding gas, and the shock wave in the gas of the source are denoted by the letters i, e, and s, respectively.

At the initial time $t = t_1$ the velocity of the motion of the contact surface is $V_i = u_1$. The gas of the source acts on the surrounding gas like an expanding cylindrical or spherical piston and, since according to the conditions of the problem $u_1 > 0$ is reached instantly, the compression wave in the surrounding gas at the moment t_1 will be centered. Therefore it must be assumed that the compression of the surrounding gas from the very start of the discharge will take place with the participation of a shock wave (e).

With an increase in r_1 the mass of the surrounding gas displaced by the "piston" and set into motion by it and the reaction of the surrounding pressure increase, whereas the motive impulse supplied by the source per unit time remains constant. Therefore the motion of the surface i slows down with time, which in turn impedes the free expansion of the gas escaping from the source and leads to its accumulation in front of the surface i and to compression. In the initial period of discharge the process of compression will be isentropic, since at $t = t_1$ the velocity of the surface i is equal to the velocity of the gas "accumulating" at it and the difference in velocities increases continuously with time. With time the wave of isentropic compression changes into a shock wave, whose path of motion is shown by the solid curve s; the dashed section of the curve corresponds to the stage of isentropic compression. The flow of the gas in the region (r_1, r_s) (under the solid curve) is not disturbed and takes place just as if the gas were escaping into a vacuum.

As $t \to \infty$ we have $r_i, r_e \to \infty$. As this happens $V_i \to 0$ and the wave e degenerates into a weak disturbance. At the same time the pressure p_∞ of the surrounding gas becomes determining in the region (r_s, r_i) as a result of which the shock wave s approaches a certain fixed position $r_s = r_{ss}$ determined by the value $\sim r_1 (p_{01}/p_\infty)^{1/\nu}$ [3].

Henceforth we will use the following terminology (see Fig. 1): I, initial stage of expansion ($r_e - r_i \ll r_i$; $r_i - r_s \ll r_i$; $r_i \ll r_{ss}$); II, intermediate stage of expansion; III, final stage of expansion ($t \to \infty$; $r_i \to \infty$; $r_e \to \infty$; $r_s \to r_{ss}$).

3. Principal Laws of Motion of the Surfaces of a Powerful Explosion in the Initial Stage of Expansion of a Gas into a Flooded Space

We shall confine ourselves to an examination, within the framework of the theory of an ideal fluid, of the model problem of the expansion of a gas from a suddenly turned on steady source with $M_1^2 \gg 1$. We shall take the conditions in the flooded space as typical for practice: $\rho_\infty \ll \rho_1$; $T_\infty \ll T_{01}$. In the limiting case of expansion of a gas into a vacuum the problem has a simple solution: the gas (in particular, its leading front, i.e., the contact surface) moves with an almost constant velocity $u \simeq u_{max} \simeq u_1$, while the distributions of p, ρ, and T in the region (r_1, r_i) are close to those for steady expansion [1].

Let us determine the motion of the contact surface, starting from the law of conservation of momentum. Using considerations of the theory of a thin compresed layer [4, 5], we will assume that in the initial stage of expansion the mass of gas concentrated in the compressed layer (r_s, r_e) is moving on the average with the velocity $V_i = dr_i/dt$. Since according to the conditions of the problem $M_1^2 \gg 1$ and the thickness of the compressed layer is small, the effect of the pressure due to the widening of the stream tubes on the motion of the gas can be neglected. In addition, one can also neglect the counterpressure p_∞, since for small enough T_∞ (or, to be more precise, for $c_\infty^2 \ll V_i^2$) the retardation of the escaping gas will take place mainly

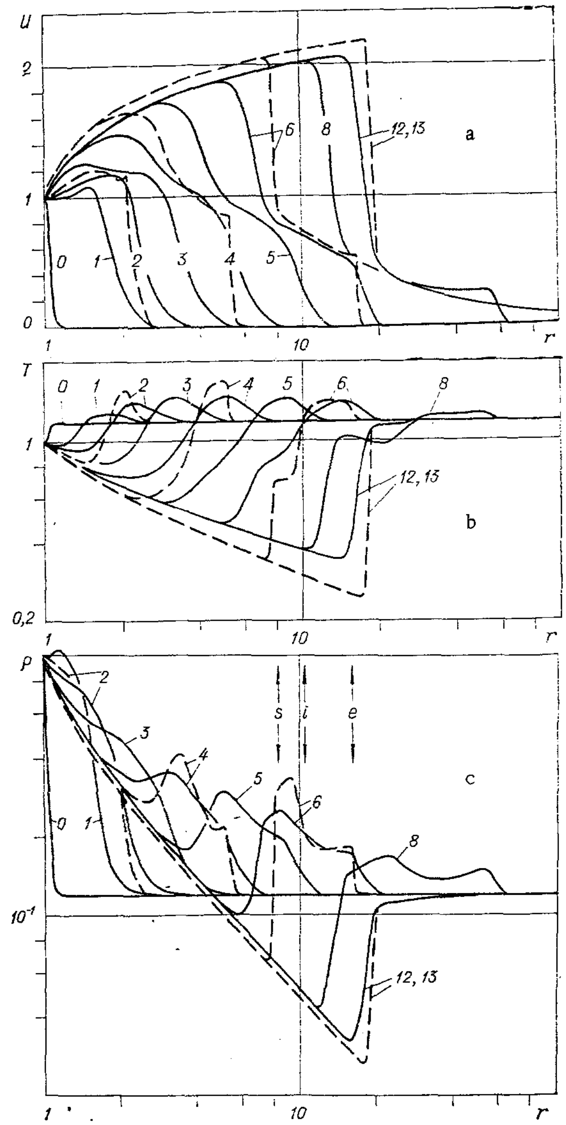

Fig. 2

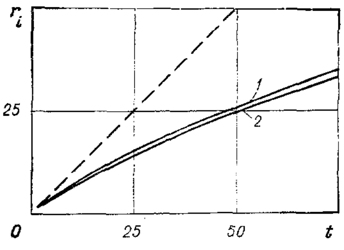

Fig. 3

through the setting into motion of the displaced surrounding gas. With allowance for this the equation of conservation of momentum is written in the form

$$\frac{d}{dt'}\left[(m'_{s,i}+m'_{i,e})\frac{dr'_i}{dt'}+2\nu\pi\int\limits_{r'_1}^{r'_s}\rho'u'r'^{\nu}dr'\right]=2\nu\pi\rho'_1u_1'^{2}r_1'^{\nu}, \tag{3.1}$$

where the prime denotes a dimensional variable, while $m'_{s,i}$ and $m'_{i,e}$ denote the mass of gas in the regions (r_s, r_i) and (r_i, r_e), respectively.

It is clear that $m'_{i,e}$ is approximately equal to the mass of displaced surrounding gas. In particular, for $r'_i \gg r'_1$

$$m'_{i,e}=\frac{2\nu}{\nu+1}\pi\rho'_{\infty}r_i'^{\nu+1}. \tag{3.2}$$

We can use the equation of conservation of mass of the escaping gas to determine the value $m'_{s,i}$. When $t' \gg t'_1$ we have approximately

$$m'_{s,i}=2\nu\pi\rho'_1u'_1r_1'^{\nu}t'-2\nu\pi\int\limits_{r'_1}^{r'_s}\rho'r'^{\nu}dr'. \tag{3.3}$$

TABLE 1

		Experiment			Theory
		$\varkappa=9/7$	$\varkappa=7/5$	$\varkappa=5/3$	$\varkappa=9/7;\ 7/5;\ 5/3$
$\nu=1$	n_i	0,68	0,71	0,65	0,67
	n_e	0,71	0,71	0,69	
$\nu=2$	n_i	—	0,54	0,52	0,5
	n_e	—	0,66	0,63	

In the region (r_1, r_s) the flow of the gas takes place just as if the gas were expanding into a vacuum. In particular, when $M_1^2 \gg 1$ we have $u' \simeq u'_1$ and

$$\rho' r'^{\nu} = \rho'_1 r_1'^{\nu}, \tag{3.4}$$

which we can use for calculating the integrals entering into (3.1) and (3.3). Replacing, in addition, the upper limits of integration in these integrals with the close value r_i, after the substitution of (3.2) and (3.3) into (3.1) and twofold integration we obtain the law of motion of the contact surface:

$$(r_i - t)^2 = b r_i^{\nu+2} + C_1 t + C_2, \quad b \equiv \frac{2}{(\nu+1)(\nu+2)} \rho_\infty, \tag{3.5}$$

where $r_i = r'_i/r'_1$; $t = t'u'_1/r'_1$; $\rho_\infty = \rho'_\infty/\rho'_1$. When r_i, $t = 0$ (1) the estimate $C_1, C_2 \geq 0$ (ρ_∞) follows from (3.5) and the first-order differential equation for the constants of integration which corresponds to it. Therefore, for r_i, $t \gg 1$ we finally have

$$t = r_i + \sqrt{b}\, r_i^{\frac{\nu+2}{2}}. \tag{3.6}$$

For $r_i \ll b^{-1/\nu}$ Eq. (3.6) gives $r_i = t$, which corresponds to the solution for the expansion of a gas into a vacuum [1].

The effect of the flooded space begins to be felt when $r_i = 0(b^{-1/\nu})$. Let us examine the solution obtained when $r_i \gg b^{-1/\nu}$, where

$$r_i = b^{-1/\nu+2}\, t^{2/\nu+2}. \tag{3.7}$$

Returning to dimensional variables in (3.7)

$$r'_i = \left[\frac{(\nu+1)(\nu+2)}{2} \frac{\rho'_1 u_1'^2 r_1'^{\nu}}{\rho'_\infty}\right]^{\frac{1}{\nu+2}} t'^{\frac{2}{\nu+2}}, \tag{3.8}$$

we see that the motion of the contact surface for each $\nu = 1, 2$ is determined by only two values: ρ'_∞ and the impulse of the source per unit time $I'_1 = 2\pi\nu\rho'_1 u'^2_1 r'^{\nu}_1$. When the pressure p'_∞ is neglected the state of the gas in the surrounding space is determined by the single nonzero dimensional value ρ'_∞. Therefore, the very process of flow of the gas in the region (r_i, ∞) is determined by only two dimensional values: I'_1 and ρ'_∞. Since their dimensionalities are independent, in the region (r_i, ∞) there is a self-similar solution of the problem which depends on ν, $\varkappa$, and the single variable [6]

$$\lambda = \left(\frac{I'_1}{\rho'_\infty}\right)^{-\frac{1}{2+\nu}} r' t'^{-\frac{2}{\nu+2}}. \tag{3.9}$$

In this case certain fixed values of λ correspond to the surfaces of a powerful explosion.

This solution comes down to the well-known solution of the problem of the displacement of a gas by an expanding cylindrical or spherical piston moving in accordance with the power law $r'_i = C' t'^{n+1}$ of (3.8). In particular, we have

$$r_e(t) = \frac{\lambda_e}{\lambda_i} r_i(t); \quad \frac{\lambda_e}{\lambda_i} = a(n, \varkappa, \nu), \tag{3.10}$$

where $n = -\nu/(\nu+2)$. For $\varkappa = 1.4$ the numerical values of $a = a(n, \varkappa, \nu)$ are given in [7].

Equation (3.10) can also be used to estimate the value of $r_e(t)$ when $r_i \leq 0(b^{-1/\nu})$, since the value of a depends weakly on the exponent n [which cannot be said about the distribution itself of the parameters in the region (r_i, r_e)]. For example, for $\varkappa=1.4$ with the maximum variation in n for (3.6) [from 0 to $-\nu/(\nu+2)$] the value of a for $\nu=1$ and $\nu=2$ varies within limits of 5% about some $a_{av}=1.13$ [7].

Let us find the approximate law of motion of the shock wave s. Assuming that $\rho' \simeq \rho'_s(\varkappa+1)/(\varkappa-1)$ in the region (r_s, r_i), from (3.3) with the help of (3.4) we obtain

$$\frac{\varkappa+1}{\varkappa-1}\frac{r_i^{\nu+1}-r_s^{\nu+1}}{r_s^{\nu}}=(\nu+1)(t-r_s),$$

where $t=t'u'_1/r'_1$; $r_\alpha=r'_\alpha/r'_1$. Or since $r_i-r_s \ll r_i$

$$r_s=\frac{\varkappa+1}{2}\left(r_i-\frac{\varkappa-1}{\varkappa+1}t\right). \tag{3.11}$$

For the expansion of a gas into a vacuum, when $r_i=t$, (3.11) gives the correct result: $r_s(t)=r_i(t)$.

Note that Eqs. (3.6), (3.10), and (3.11), which determine the laws of motion of the surfaces i, e, and s of a powerful explosion in the initial stage of expansion, allow one to change to the new variables

$$\tau=\left(\frac{\rho'_\infty}{\rho'_1}\right)^{1/\nu}\frac{t'u'_1}{r'_1};\quad \xi_\alpha=\left(\frac{\rho'_\infty}{\rho'_1}\right)^{1/\nu}\frac{r'_\alpha}{r'_1};\quad \alpha=i,e,s,$$

in which the motion of these surfaces will be given by single equations for different ρ'_∞/ρ'_1.

The results of the analysis conducted above allow one to explain certain regularities of the motion of the surfaces of a powerful explosion during the discharge of shock-heated gas from a flat slot (analog: $\nu=1$) and from a round opening ($\nu=2$) [2].

We should note the following in advance. The condition $r_i \gg b^{-1/\nu}$ is equivalent to the condition that the mass $m'_{i,e}(\sim\rho'_\infty r_i^{\nu+1}(t))$ of displaced surrounding gas is much greater than the mass $m'_{1,e}(\sim\rho'_1 u'_1 r'^{\nu}_1 t')$ of gas coming through the source. This gives a physical explanation for the fact that the law of motion $r'_i=r'_i(t)$ of the contact surface is determined here only by the values ν, I'_1, and ρ'_∞.

It is reasonable to assume that in the case of the expansion of a gas from a source with $M_1=1$, as occurs in the experiments of [2], in the section of the initial stage of expansion where $m'_{i,e} \gg m'_{1,i}$ and $r'_i \gg r'_1$ the motion of the contact surface will be determined first of all by the two-dimensional values: $I'_1=2\nu\pi(\rho_i u_i^2+p'_1)r_1'^{\nu}$ and ρ'_∞. Then the arguments which led to (3.9)-(3.10) above are valid here.

In [2] the laws of motion of the front of escaping gas ($\alpha=i$) and of the shock wave in the surrounding gas ($\alpha=e$) were determined experimentally and represented in the form of the functions $x_\alpha=C_\alpha t^{n_\alpha}(x'\simeq r')$. An analysis of the experimental conditions shows that the condition $m'_{i,e} \gg m'_{1,i}$ is satisfied in almost the entire initial stage of expansion. A comparison of the values of the exponent n_α with the theoretical values is given in Table 1.

Numerical values of a from (3.10) are known for $\varkappa=1.4$ [7]. Therefore the value of x_e/x_i can be compared with the theoretical value. For the discharge of $N_2(\varkappa=1.4)$ from a slot, where $n_e=n_i\simeq 0.67$, we have $x_e/x_i=1.19$ and $r_e/r_i=1.16$.

4. Numerical Method of Solving the Problem for a Viscous Thermally Conducting Gas

The unsteady radial flow of a viscous, thermally conducting, compressible, ideal gas is described by the following system of equations [8]:

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial r}+\frac{1}{\varkappa M_1^2}\frac{\partial p}{\partial r}=\frac{1}{\mathrm{Re}_1}\left\{\frac{4}{3}\frac{\partial}{\partial r}\left[\frac{\mu}{r^{\nu}}\frac{\partial}{\partial r}(r^{\nu}u)-2\nu\frac{u}{r}\frac{\partial\mu}{\partial r}\right]\right\}; \tag{4.1}$$

$$\rho\frac{\partial T}{\partial t}+\rho u\frac{\partial T}{\partial r}-\frac{\varkappa-1}{\varkappa}\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right)=\frac{1}{\mathrm{Re}_1}\left\{\frac{1}{\sigma}\frac{1}{r^{\nu}}\frac{\partial}{\partial r}\left(r^{\nu}\mu\frac{\partial T}{\partial r}\right)+\right.$$
$$\left.+\frac{4}{3}(\varkappa-1)M_1^2\mu\left[\left(\frac{\partial u}{\partial r}\right)^2-\nu\frac{u}{r}\frac{\partial u}{\partial r}+\left(\frac{u}{r}\right)^2\right]\right\}; \tag{4.2}$$

$$\frac{\partial\rho}{\partial t}+\frac{1}{r^{\nu}}\frac{\partial}{\partial r}(r^{\nu}\rho u)=0; \tag{4.3}$$

$$p=\rho T, \tag{4.4}$$

where all the values are normalized to the corresponding values at $r' = r'_1$; $t = t'u'_1/r'_1$; $Re_1 = \rho'_1 u'_1 r'_1/\mu'_1$; $M_1 = u'_1/c'_1$; the Prandtl number σ and the ratio of heat capacities $\varkappa$ are assumed to be constant.

Stating from the general statement of the problem and keeping in mind the nature of the system of equations (4.1)-(4.4), we can write the boundary conditions in the following form:

when $t=1$ $u=1$, $T=1$, and $\rho=1$ for $r=1$; $u=0$, $T=T_\infty$, and $\rho=\rho_\infty$ for $1<r\leq\infty$;
when $t>1$ $u=1$, $T=1$, and $\rho=1$ for $r=1$; $u=0$, $T=T_\infty$ for $r=\infty$.

To find the values of u, T, and ρ in the region $(1, \infty)$ when $t>1$ we propose to use Eqs. (4.1)-(4.3), respectively. By eliminating the pressure p from (4.1)-(4.2) with the help of (4.4) and noting the derivative $\partial\rho/\partial t$ which appears in (4.2) in this case with the help of the equation of continuity (4.3), we can write the system of equations in the following linear form:

$$\frac{\partial u}{\partial t} + A_1\frac{\partial^2 u}{\partial y^2} + A_2\frac{\partial u}{\partial y} + A_3 u = A_4; \tag{4.5}$$

$$\frac{\partial T}{\partial t} + B_1\frac{\partial^2 T}{\partial y^2} + B_2\frac{\partial T}{\partial y} + B_3 T = B_4; \tag{4.6}$$

$$\frac{\partial \rho}{\partial t} + C_2\frac{\partial \rho}{\partial y} + C_3\rho = C_4, \tag{4.7}$$

where $y=y(r)$ is a transformation which changes $[1,\infty] \ni r$ into $[0,1] \ni y$: A_k, B_k, and C_k are complexes remaining after the isolation of the linear section. If one considers that the terms containing the products $u(\partial u/\partial y)$ and $T(\partial T/\partial y)$ were included in the groups $A_2(\partial u/\partial y)$ and $B_2(\partial T/\partial y)$, respectively, the notation (4.5)-(4.7) uniquely determines the form of A_k, B_k, and C_k.

As the experiment of an earlier report [9] shows, to solve the problem with an acceptable accuracy and a reasonable volume of calculations one must have a finite-difference system with an accuracy no worse than $0(h^2)$ with respect to space y. In the approximation of Eqs. (3.7) and (3.8) by finite differences one can use the implicit difference system of [9], which has an approximation accuracy $0(\tau + h^2)$ and for the corresponding linear equations is stable for any finite τ/h^2, where τ and h are the steps in time and in space y, respectively. In particular, for (4.5)

$$\frac{u_i^{j+1} - u_i^j}{\tau} + A_{1i}^j\frac{u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1}}{h^2} + A_{2i}^j\frac{u_{i+1}^{j+1} - u_{i-1}^{j+1}}{2h} + A_{3i}^j u_i^{j+1} = A_{4i}^j + 0(\tau + h^2), \tag{4.8}$$

where i is the number of the partition with respect to $y (0 \leq i \leq N)$; j is the number of the time layer $(j \geq 0)$. The value of A^j_{ki} are calculated as follows: the values entering into A_k are calculated algebraically, assuming that their values are equal at the node (t^{j+1}, y_i), while the derivatives with respect to y are calculated at the point i of the j-th layer using the central differences $0(h^2)$.

Equation (4.6) is approximated in a similar way.

With an equation of the hyperbolic type of (4.7) the matter becomes somewhat more complicated (we assume that the C_k are known). The implicit difference system with the natural, in the given case right-sided, approximation of the spatial derivative leads to a spatial accuracy of only $0(h)$ [10]. Therefore, we proceed as follows. From the Taylor series expansion of ρ with respect to y in the vicinity of the point (t^{j+1}, y_i) we have

$$\left(\frac{\partial\rho}{\partial y}\right)_i^{j+1} = \frac{\rho_{i+1}^{j+1} - \rho_i^{j+1}}{h} - \left(\frac{\partial^2\rho}{\partial y^2}\right)_i^{j+1}\frac{h}{2} + 0(h^2).$$

Replacing $(\partial^2\rho/\partial y^2)_i^{j+1}$ with the corresponding value in the j-th layer, we obtain

$$\left(\frac{\partial\rho}{\partial y}\right)_i^{j+1} = \frac{\rho_{i+1}^{j+1} - \rho_i^{j+1}}{h} - \left(\frac{\partial^2\rho}{\partial y^2}\right)_i^{j}\frac{h}{2} + 0(\tau + h^2).$$

Thus, the difference system for (4.7) can be written in the form

$$\frac{\rho_i^{j+1} - \rho_i^j}{\tau} + C_{2i}^j\frac{\rho_{i+1}^{j+1} - \rho_i^{j+1}}{h} + C_{3i}^j\rho_i^{j+1} + C_{4i}^j = C_{2i}^j\left(\frac{\partial^2\rho}{\partial y^2}\right)_i^j\frac{h}{2} + 0(\tau + h^2), \tag{4.9}$$

where $(\partial^2\rho/\partial y^2)_i = (\rho_{i+1}^j - 2\rho_i^j + \rho_{i-1}^j)/h^2$, while the C_{ki}^j are calculated analogously to the A_{ki}^j and B_{ki}^j. It is not hard to verify [10] that when $C_k = \text{const}$ (i.e., for a linear equation) the sytem (4.9) is stable for any finite τ/h.

The values of u_i^{j+1} for $j \ge 0$ were determined by the trial run method [10] from the system of equations $u_0^{j+1} = 1$ and (4.8) for $1 \le i \le N-1$ and $u_i^{j+1} = 0$. The T_i^{j+1} were calculated analogously. The values of ρ_i^{j+1} were determined from (4.9) with the initial condition $\rho_0^{j+1} = 1$. The value of ρ_N^j needed for this when $j \ge 1$ was determined by extrapolation from the $\rho^j_i (N-3 \le i \le N-1)$. In the calculations conducted the ρ^j_N coincided with $\rho^0_N = \rho_\infty$ with an accuracy of five significant figures.

The function $y = (2/\pi)\arctan[(r-1)/l]$, where $l = 0(r_{ss})$, we used for $y = y(r)$ in the calculations. The step h in space was taken as uniform while the value of the step τ in time, in connection with the fact that the time rate of development of the process varies quite considerably in the course of time, was taken from the equation

$$\tau^{j+1}A^j = \tau^1 A^0, \quad A^j = \max_{1<i<N-1}\left\{\left(\frac{\partial \ln u}{\partial t}\right)_i^j, \ \left(\frac{\partial \ln T}{\partial t}\right)_i^j, \ \left(\frac{\partial \ln \rho}{\partial t}\right)_i^j\right\}.$$

The process was considered as established when $A^j \le \varepsilon$. The values of τ^0, h, and ε were selected experimentally in such a way that upon their fivefold variation the distribution of the parameters at similar times did not differ by more than 2-3%.

5. Results of a Numerical Solution of the Problem for a Viscous Thermally Conducting Gas

The distributions of the stream parameters for the expansion of a gas from a suddenly turned on cylindrical source ($\nu = 1$) with $M_1 = M_* \equiv 1$ into a medium with $\rho_\infty = 0.12$ and $T_\infty = T_{0*} = 1.2$ are given in Fig. 2 as an example illustrating the development of the flow of a viscous thermally conducting gas. Here $\varkappa = 7/5$, $\sigma = 3/4$, and $\mu = T$.

The distributions of the parameters corresponding to the time $t_0 = 1$ are denoted by the number 0. These distributions, which represent smoothed discontinuities, were given as the initial distributions in the calculations. The times $t_k = 1 + 0.32 \cdot 2^k$ are denoted by the numbers 1-13. The solid lines correspond to $Re_* = 25$ and the dashed lines to $Re_* = 200$. The arrows with the indices s, i, and e show the conditional, within the limits of 5-7%, positions of the corresponding surfaces of the powerful explosion for the time $t_6 = 11.28$ (here and afterward the term "surface" will be used in the case when discussing integral characteristics of the shock waves or the contact zone: position, drop in parameters, etc.).

The overall pattern of the flow corresponds to that presented in Part 2. The shock wave e is formed almost immediately after the beginning of the discharge of gas. For example, at the time t_2 the difference in the value of $T/\rho^{\varkappa-1}$ "before" and "after" the wave is already 30% (for $Re_* = 200$). The process of compression of the gas in front of the inner surface of the "piston" i continues to be isentropic for some time. For example, for the same Re_* at the time t_4 the behavior of ρ and T in the region from $r = 1$ to the point with the maximum value of ρ still obeys the law $T = \rho^{\varkappa-1}$ with an accuracy of 5%. At the time t_6 the shock wave s is already formed.

The flow of gas in the region $(1, r_s)$ takes place just as for expansion into a vacuum. Since ρ_∞ and p_∞ are rather large in the given case, however, the contact surface moves slower than the boundary of steady flow during the expansion of a gas into a vacuum [1]. Therefore the flow of gas is steady in the region $(1, r_s)$. It has the properties inherent to the steady radial flow during the expansion of a gas into a vacuum [9]. We note that the effect of Re_*, $\varkappa$, σ, and the dependence $\mu = \mu(T)$ are mainfested analogously for $\nu = 1$ and $\nu = 2$.

The behavior of the stream parameters in the region (r_i, r_e) in the initial stage of expansion [up to $t = 0(t_6)$] correspond qualitatively to the behavior of the parameters in the problem on the piston [7].

As $t \to \infty$ we have r_e, $r_i \to \infty$ and $r_s \to r_{ss}$. In this case $T_{i+} \to T_\infty$ as a result of the degeneration of the shock wave e into a weak disturbance. At the same time T_{i-} approaches the stagnation temperature T_{0*} of the escaping gas, in this case equal to T_∞. Consequently, $T_{i+} \to T_{i-}$. Since at the contact surface $u_{i+} = u_{i-}$ and $p_{i+} = p_{i-}$ in addition, the drop in the parameters at it disappears as $t \to \infty$ and the flow changes to a stationary mode.

It should be noted that a decrease in the overall compactness of the stream (a decrease in Re_*), which leads to a considerable change in the parameters in the entire region of flow, has a rather weak effect on the instantaneous position of the shock waves and the contact surface.

The qualitative pattern of flow for the case of the spherical ($\nu=2$) expansion of a viscous thermally conducting gas into a flooded space with $T_\infty = T_{0*}$ is fully analogous to that described above. The important difference between the cases of $\nu=1$ and $\nu=2$ appears when $T_\infty \neq T_{0*}$ and involves mainly the final stage of expansion. This question requires a special examination and will not be discussed in the present article.

A comparison of the dependences $r_i = r_i(t)$ for the initial stage of expansion of a gas for $\nu=1$ found through the approximate analysis of Part 3 [Eq. (3.6), curve 1] and from the results of a numerical calculation (curve 2) is given in Fig. 3. Here $\varkappa = 7/5$; $M_1 = 5$; $\rho'_\infty/\rho'_1 = 0.123$; $T'_\infty/T'_1 = 0.145$. To obtain better accuracy in the approximate solution the value $r'_e = ar'_i$, which corresponds to $m'_{i,e}$ in physical meaning, was inserted into (3.2) in place of r'_i in accordance with (3.10). This led to a new form of the value in (3.6): $b = \frac{2a^{\nu+1}}{(\nu+1)(\nu+2)}\rho_\infty$, $a \doteq 1.13$. The additional conditions for the numerical calculations are: $\sigma = 3/4$; $\mu = T$; $Re_1 = 400$. The dependence $r_e = t$ which corresponds to expansion into a vacuum is shown by a dashed line for clearness.

The approximate dependences $r_e(t)$ and $r_s(t)$ determined by Eqs. (3.6), (3.10) with $a = 1.13$, and (3.11) agree with an accuracy of 5-10% with those resulting fron a numerical calculation.

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NOTATION

r, radius; t, time; T, temperature; p, pressure; ρ, density; u, velocity of gas; $\varkappa$, ratio of heat capacities; M, Mach number; x, distance along axis of jet from nozzle cut; V_α, translational velocity of surface α; $m_{\alpha,\beta}$, mass of gas included in the region (r_α, r_β); c, velocity of sound; μ, dynamic viscosity.

INDICES

$\nu = 1, 2$, cylindrical and spherical symmetry; ', dimensional value; $-$ (+), left (right) side of surface of powerful explosion; ∞, conditions in flooded space; 0, conditions of adiabatic stagnation; *, sonic surface; 1, surface of source.

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